① Given $f(x) = 3x^2 - x$, find

ⓐ the slope of the curve at any point $x = a$.

ⓑ when the slope will be -16.

② Find the tangent and normal line to $f(x) = x^2 + 4x$ at $x = -3$.

$f'(x) = 2x + 4$

$f'(-3) = -2$

ⓐ $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = 3x^2 - x$

$f(x+h) = 3(x+h)^2 - (x+h)$

$= 3x^2 + 6xh + 3h^2 - x - h$

$$\frac{\cancel{3x^2} + 6x\cancel{h} + 3h^{\cancel{2}} \cancel{- x} - \cancel{h} - \cancel{3x^2} \cancel{+ x}}{\cancel{h}}$$

$\lim_{h \to 0} 6x + 3h - 1 = 6x - 1 =$ slope at any pt. $= 6a - 1 =$ derivative $f'(x)$ of $f(x)$

$6a - 1 = -16$

$a = -2.5$

$f(x) = x^2 + 4x$

$f(-3) = (-3)^2 + 4(-3)$
$= -3$

$f'(x) = 2x + 4$

$(-3, -3)$

$f'(-3) = -2$

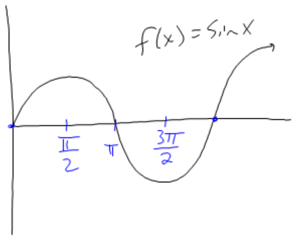
pt

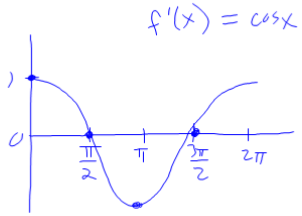
tangent line $y = m(x - x_1) + y_1$

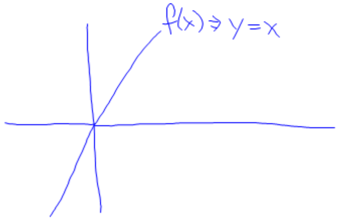
$y = -2(x + 3) - 3$

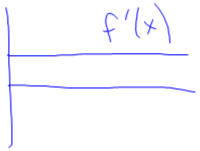
Normal $y = \frac{1}{2}(x + 3) - 3$

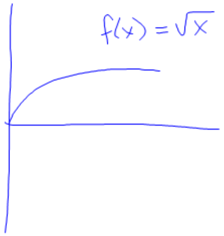
25

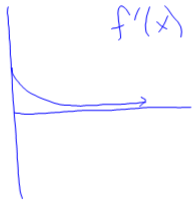

$f(x) = \sin x$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

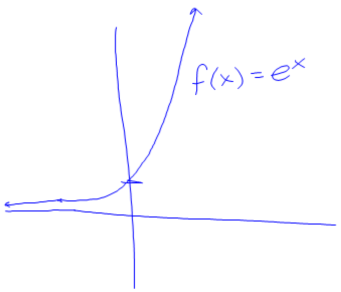

$f'(x) = \cos x$
1
0
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

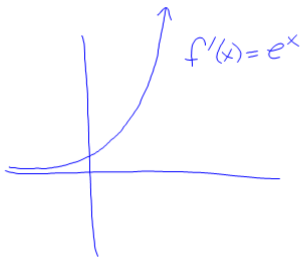

$f(x) \Rightarrow y = x$

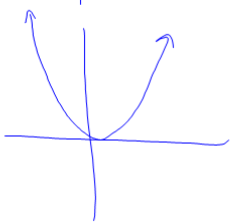

$f'(x)$

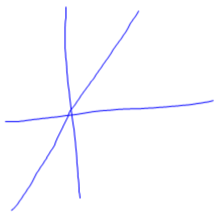

$f(x) = \sqrt{x}$


$f'(x)$

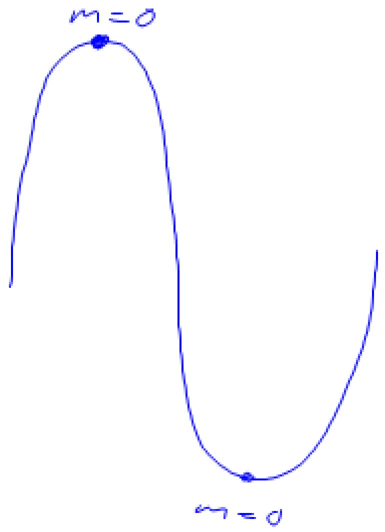

$f(x) = e^x$

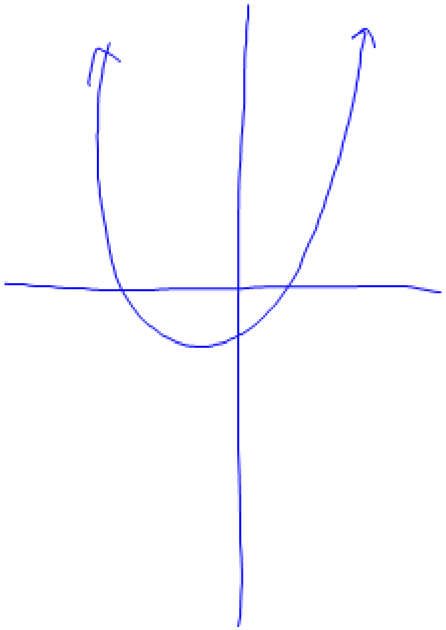

$f'(x) = e^x$





Do 3.1 #22





17 pt. $(2, 3)$

Slope $= f'(x)$

Slope at $2 = 5$

$f'(2) = 5$

$y = m(x - x_1) + y_1$

tangent $\boxed{\begin{aligned} y &= 5(x-2)+3 \\ y &= 5x - 7 \end{aligned}}$

Normal $y = -\frac{1}{5}(x-2) + 3$

$y = -\frac{1}{5}x + 3.4$

① $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \Longrightarrow \lim_{h\to 0} \frac{\frac{x}{x}\cdot\frac{1}{x+h} - \frac{1}{x}\cdot\frac{x+h}{x+h}}{h}$$

$$\lim_{h\to 0} \frac{\frac{\cancel{x} - \cancel{x} - h}{x(x+h)}}{h} = \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-1}{x(x+h)} = \boxed{-\frac{1}{x^2}}$$

5) $f(x) = \frac{1}{x}$

$f(a) = \frac{1}{a}$

$$\frac{f(x) - f(a)}{x - a} \Rightarrow \frac{\frac{a}{a}\cdot\frac{1}{x} - \frac{1}{a}\cdot\frac{x}{x}}{x - a}$$

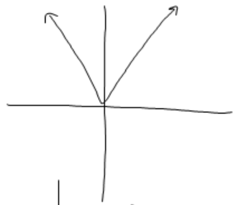
$$\frac{\frac{a - x}{ax}}{x - a} \Rightarrow \frac{a - x}{ax} \cdot \frac{1}{x - a} \Rightarrow \frac{-\cancel{(x - a)}}{ax} \cdot \frac{1}{\cancel{x - a}}$$

$$= \frac{-1}{ax}$$

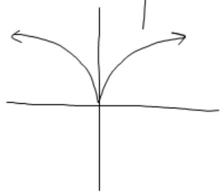
$$\lim_{x \to a} -\frac{1}{ax} = \boxed{-\frac{1}{x^2}}$$

Not Differentiable if there is a

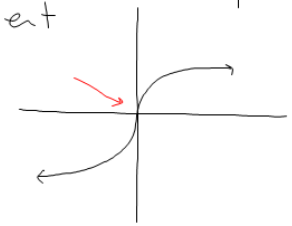
(a) corner $f(x) = |x|$



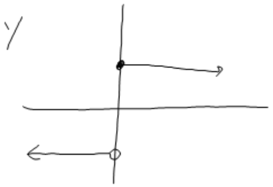
(b) cusp $f(x) = x^{2/3}$



(c) Infinite tangent



(d) Discontinuity



Does continuity imply differentiability?

NO! corner
cusp
infinite tangent line

Does differentiability imply continuity?

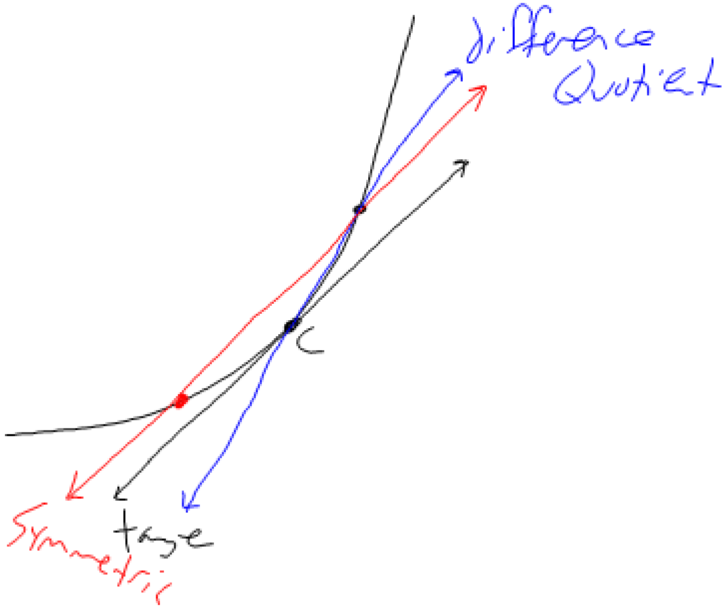
yes!

Intermediate Value Theorem for derivatives

$f'(a) = m$ $f'(b) = n$ and $f'(x)$ is differentiable at all points between a & b

$f'(x)$ takes on all values between m & n

Symmetric Difference Quotient - often better



$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(x+h)-f(x-h)}{2h}$$

Numeric approximation for small h value

$f(x)=x^2$ $\quad h=0.01$ $\quad x=10$

$f(x) = x^2 \quad h = 0.01 \quad \text{at } x = 10$

$$\frac{((x+h)^2) - (x^2)}{h} \qquad \frac{(x+h)^2 - (x-h)^2}{2h}$$

$$\frac{(10+0.01)^2 - 10^2}{0.01} \qquad \frac{(10+0.01)^2 - (10-0.01)^2}{2(0.01)}$$

$$\frac{100.2001 - 100}{0.01} \qquad \frac{100.2001 - 99.8001}{0.02}$$

$$20.01 \qquad \frac{.4}{0.02} = 20$$

Sect. 3.2

#1, 4, 5, 8, 10, 12-39 (mult. of 3)